

**BEMIDJI AREA SCHOOLS**  
**Outcomes in Mathematics – Analysis I**

Strand	Standard	No.	Benchmark
Algebra	Understand the concept of function, and identify important features of functions and other relations using symbolic and graphical methods.	9.2.1.1	Understand the definition of a function. Use functional notation and evaluate a function at a given point in its domain. <i>For example: If <math>f(x) = \frac{1}{x^2 - 3}</math>, find <math>f(-4)</math>.</i>
		9.2.1.2	Distinguish between functions and other relations defined symbolically, graphically or in tabular form.
		9.2.1.3	Find the domain of a function defined symbolically, graphically or in a real-world context. <i>For example: The formula <math>f(x) = \pi x^2</math> can represent a function whose domain is all real numbers, but in the context of the area of a circle, the domain would be restricted to positive <math>x</math>.</i>
		9.2.1.4	Obtain information and draw conclusions from graphs of functions and other relations. <i>For example: If a graph shows the relationship between the elapsed flight time of a golf ball at a given moment and its height at that same moment, identify the time interval during which the ball is at least 100 feet above the ground.</i>
	Understand the concept of function, and identify important features of functions and other relations using symbolic and graphical methods where appropriate.	9.2.1.5	Identify the vertex, line of symmetry and intercepts of the parabola corresponding to a quadratic function, using symbolic and graphical methods, when the function is expressed in the form $f(x) = ax^2 + bx + c$ , in the form $f(x) = a(x - h)^2 + k$ , or in factored form.
		9.2.1.6	Identify intercepts, zeros, maxima, minima and intervals of increase and decrease from the graph of a function.
		9.2.1.7	Understand the concept of an asymptote and identify asymptotes for exponential functions and reciprocals of linear functions, using symbolic and graphical methods.
		9.2.1.8	Make qualitative statements about the rate of change of a function, based on its graph or table of values. <i>For example: The function <math>f(x) = 3^x</math> increases for all <math>x</math>, but it increases faster when <math>x &gt; 2</math> than it does when <math>x &lt; 2</math>.</i>
		9.2.1.9	Determine how translations affect the symbolic and graphical forms of a function. Know how to use graphing technology to examine translations. <i>For example: Determine how the graph of <math>f(x) =  x - h  + k</math> changes as <math>h</math> and <math>k</math> change.</i>

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Algebra	Recognize linear, quadratic, exponential and other common functions in real-world and mathematical situations; represent these functions with tables, verbal descriptions, symbols and graphs; solve problems involving these functions, and explain results in the original context.	9.2.2.1	Represent and solve problems in various contexts using linear and quadratic functions. <i>For example:</i> Write a function that represents the area of a rectangular garden that can be surrounded with 32 feet of fencing, and use the function to determine the possible dimensions of such a garden if the area must be at least 50 square feet.
		9.2.2.2	Represent and solve problems in various contexts using exponential functions, such as investment growth, depreciation and population growth.
		9.2.2.3	Sketch graphs of linear, quadratic and exponential functions, and translate between graphs, tables and symbolic representations. Know how to use graphing technology to graph these functions.
	Recognize linear, quadratic, exponential and other common functions in real-world and mathematical situations; represent these functions with tables, verbal descriptions, symbols and graphs; solve problems involving these functions, and explain results in the original context.	9.2.2.6	Sketch the graphs of common non-linear functions such as $f(x)=\sqrt{x}$ , $f(x)= x $ , $f(x)=\frac{1}{x}$ , $f(x)=x^3$ , and translations of these functions, such as $f(x)=\sqrt{x-2}+4$ . Know how to use graphing technology to graph these functions.
		9.2.3.1	Evaluate polynomial and rational expressions and expressions containing radicals and absolute values at specified points in their domains.
	Generate equivalent algebraic expressions involving polynomials and radicals; use algebraic properties to evaluate expressions.	9.2.3.2	Add, subtract and multiply polynomials; divide a polynomial by a polynomial of equal or lower degree.
9.2.3.3		Factor common monomial factors from polynomials, factor quadratic polynomials, and factor the difference of two squares. <i>For example:</i> $9x^6 - x^4 = (3x^3 - x^2)(3x^3 + x^2)$ .	
9.2.3.5		Check whether a given complex number is a solution of a quadratic equation by substituting it for the variable and evaluating the expression, using arithmetic with complex numbers. <i>For example:</i> The complex number $\frac{1+i}{2}$ is a solution of $2x^2 - 2x + 1 = 0$ , since $2\left(\frac{1+i}{2}\right)^2 - 2\left(\frac{1+i}{2}\right) + 1 = i - (1+i) + 1 = 0$ .	

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Algebra	Generate equivalent algebraic expressions involving polynomials and radicals; use algebraic properties to evaluate expressions.	9.2.3.6	Apply the properties of positive and negative rational exponents to generate equivalent algebraic expressions, including those involving $n^{\text{th}}$ roots.  <i>For example:</i> $\sqrt{2} \times \sqrt{7} = 2^{\frac{1}{2}} \times 7^{\frac{1}{2}} = 14^{\frac{1}{2}} = \sqrt{14}$ . Rules for computing directly with radicals may also be used: $\sqrt{2} \times \sqrt{x} = \sqrt{2x}$ .
		9.2.3.7	Justify steps in generating equivalent expressions by identifying the properties used. Use substitution to check the equality of expressions for some particular values of the variables; recognize that checking with substitution does not guarantee equality of expressions for all values of the variables.
	Represent real-world and mathematical situations using equations and inequalities involving linear, quadratic, exponential, and $n^{\text{th}}$ root functions. Solve equations and inequalities symbolically and graphically. Interpret solutions in the original context.	9.2.4.1	Represent relationships in various contexts using quadratic equations and inequalities. Solve quadratic equations and inequalities by appropriate methods including factoring, completing the square, graphing and the quadratic formula. Find non-real complex roots when they exist. Recognize that a particular solution may not be applicable in the original context. Know how to use calculators, graphing utilities or other technology to solve quadratic equations and inequalities.  <i>For example:</i> A diver jumps from a 20 meter platform with an upward velocity of 3 meters per second. In finding the time at which the diver hits the surface of the water, the resulting quadratic equation has a positive and a negative solution. The negative solution should be discarded because of the context.
		9.2.4.2	Represent relationships in various contexts using equations involving exponential functions; solve these equations graphically or numerically. Know how to use calculators, graphing utilities or other technology to solve these equations.
		9.2.4.3	Recognize that to solve certain equations, number systems need to be extended from whole numbers to integers, from integers to rational numbers, from rational numbers to real numbers, and from real numbers to complex numbers. In particular, non-real complex numbers are needed to solve some quadratic equations with real coefficients.
	Solve real-world and mathematical geometric problems using algebraic methods.	9.3.4.5	Know the equation for the graph of a circle with radius $r$ and center $(h,k)$ , $(x - h)^2 + (y - k)^2 = r^2$ , and justify this equation using the Pythagorean Theorem and properties of translations.
	Data Analysis & Probability	Display and analyze data; use various measures associated with data to draw conclusions, identify trends and describe relationships.	9.4.1.3

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**The student will:**

1. Know the six trigonometric functions defined for an angle in a right triangle.
2. Given the coordinates of a point on the terminal side of an angle in standard position in the xy-plane, find the values of the trigonometric functions.
3. Convert between degrees and radian measures.
4. Solve applied problems about triangles using the law of sines including the ambiguous case.
5. Solve applied problems about triangles using the law of cosines.
6. Graph the functions of the form  $A\sin(Bt + C)$ ,  $A\cos(Bt + C)$ , and  $A\tan(Bt + C)$  and know the meaning of the terms frequency, amplitude, phase shift and period.
7. Simplify trigonometric expressions using identities and verify simple trigonometric identities including  $\sin^2 x + \cos^2 x = 1$ , sum, difference, double angle and half-angle formulas for sine and cosine.
8. Find all the solutions of a trigonometric equation on various intervals.
9. Know and be able to use the definitions of the inverse trigonometric functions and related methods to solve problems such as find  $\cos(x)$  and  $\tan(x)$  given the value of  $\sin x$  and the quadrant containing the terminal side.
10. Compute the values of the trigonometric functions.
11. Apply the trigonometric functions to solve right triangles.
12. Simplify trigonometric expressions using trigonometric identities.
13. Graph the sine and cosine functions.
14. Solve problems using radian measure of angles.
15. Solve problems involving oblique triangles.
16. Solve problems involving angles of depression and elevation.
17. Utilize different area formulas to find the area of triangles.
18. Apply the fundamental identities, reciprocal identities, co-function identities and Pythagorean identities in formal proofs.