BEMIDJI AREA SCHOOLS
Outcomes in Mathematics

	Strand	Standard	No.	Benchmark
7	Data Analysis & Probability	Display and interpret data in a variety of ways, including circle graphs and histograms.	7.4.2.1	Use reasoning with proportions to display and interpret data in circle graphs (pie charts) and histograms. Choose the appropriate data display and know how to create the display using a spreadsheet or other graphing technology.
		Calculate probabilities and reason about probabilities using proportions to solve real-world and mathematical problems.	7.4.3.1	Use random numbers generated by a calculator or a spreadsheet or taken from a table to simulate situations involving randomness, make a histogram to display the results, and compare the results to known probabilities. *For example*: Use a spreadsheet function such as RANDBETWEEN(1, 10) to generate random whole numbers from 1 to 10, and display the results in a histogram.
			7.4.3.2	Calculate probability as a fraction of sample space or as a fraction of area. Express probabilities as percents, decimals and fractions. *For example*: Determine probabilities for different outcomes in game spinners by finding fractions of the area of the spinner.
			7.4.3.3	Use proportional reasoning to draw conclusions about and predict relative frequencies of outcomes based on probabilities. *For example*: When rolling a number cube 600 times, one would predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.
8	Number & Operation	Read, write, compare, classify and represent real numbers, and use them to solve problems in various contexts.	8.1.1.1	Classify real numbers as rational or irrational. Know that when a square root of a positive integer is not an integer, then it is irrational. Know that the sum of a rational number and an irrational number is irrational, and the product of a non-zero rational number and an irrational number is irrational. *For example*: Classify the following numbers as whole numbers, integers, rational numbers, irrational numbers, recognizing that some numbers belong in more than one category: $\frac{6}{3}$, $\frac{3}{6}$, $3.\overline{6}$, $\frac{\pi}{2}$, $-\sqrt{4}$, $\sqrt{10}$, -6.7.
			8.1.1.2	Compare real numbers; locate real numbers on a number line. Identify the square root of a positive integer as an integer, or if it is not an integer, locate it as a real number between two consecutive positive integers. *For example*: Put the following numbers in order from smallest to largest: 2, $\sqrt{3}$, – 4, – 6.8, $-\sqrt{37}$. *Another example*: $\sqrt{68}$ is an irrational number between 8 and 9.
8	Number & Operation	Read, write, compare, classify and represent real numbers, and use them to solve problems in various contexts.	8.1.1.3	Determine rational approximations for solutions to problems involving real numbers. *For example*: A calculator can be used to determine that $\sqrt{7}$ is approximately 2.65. *Another example*: To check that $1\frac{5}{12}$ is slightly bigger than $\sqrt{2}$, do the calculation $\left(1\frac{5}{12}\right)^2 = \left(\frac{17}{12}\right)^2 = \frac{289}{144} = 2\frac{1}{144}$. *Another example*: Knowing that $\sqrt{10}$ is between 3 and 4, try squaring numbers like 3.5, 3.3, 3.1 to determine that 3.1 is a reasonable rational approximation of $\sqrt{10}$.

	Strand	Standard	No.	Benchmark
			8.1.1.4	Know and apply the properties of positive and negative integer exponents to generate equivalent numerical expressions. *For example*: $3^2 \times 3^{(-5)} = 3^{(-3)}\left(\frac{1}{3}\right)^3 = \frac{1}{27}$.
			8.1.1.5	Express approximations of very large and very small numbers using scientific notation; understand how calculators display numbers in scientific notation. Multiply and divide numbers expressed in scientific notation, express the answer in scientific notation, using the correct number of significant digits when physical measurements are involved. *For example*: $(4.2\times10^4)\times(8.25\times10^3)=3.465\times10^8$, but if these numbers represent physical measurements, the answer should be expressed as 3.5×10^8 because the first factor, 4.2×10^4, only has two significant digits.
	Algebra	Understand the concept of function in real-world and mathematical situations, and distinguish between linear and non-linear functions.	8.2.1.1	Understand that a function is a relationship between an independent variable and a dependent variable in which the value of the independent variable determines the value of the dependent variable. Use functional notation, such as $f(x)$, to represent such relationships. *For example*: The relationship between the area of a square and the side length can be expressed as $f(x)=x^2$. In this case, $f(5)=25$, which represents the fact that a square of side length 5 units has area 25 units squared.
			8.2.1.2	Use linear functions to represent relationships in which changing the input variable by some amount leads to a change in the output variable that is a constant times that amount. *For example*: Uncle Jim gave Emily $50 on the day she was born and $25 on each birthday after that. The function $f(x)=50+25x$ represents the amount of money Jim has given after x years. The rate of change is $25 per year.
8	Algebra	Understand the concept of function in real-world and mathematical situations, and distinguish between linear and non-linear functions.	8.2.1.3	Understand that a function is linear if it can be expressed in the form $f(x)=mx+b$ or if its graph is a straight line. *For example*: The function $f(x)=x^2$ is not a linear function because its graph contains the points (1,1), (-1,1) and (0,0), which are not on a straight line.
			8.2.1.4	Understand that an arithmetic sequence is a linear function that can be expressed in the form $f(x)=mx+b$, where $x = 0, 1, 2, 3,\ldots.$ *For example*: The arithmetic sequence 3, 7, 11, 15, …, can be expressed as $f(x) = 4x + 3$.
			8.2.1.5	Understand that a geometric sequence is a non-linear function that can be expressed in the form $f(x)=ab^x$, where $x = 0, 1, 2, 3,\ldots.$ *For example*: The geometric sequence 6, 12, 24, 48, … , can be expressed in the form $f(x) = 6(2^x)$.
		Recognize linear functions in real-world and	8.2.2.1	Represent linear functions with tables, verbal descriptions, symbols, equations and graphs; translate from one representation to another.

	Strand	Standard	No.	Benchmark
		mathematical situations; represent linear functions and other functions with tables, verbal descriptions, symbols and graphs; solve problems involving these functions and explain results in the original context.	8.2.2.2	Identify graphical properties of linear functions including slopes and intercepts. Know that the slope equals the rate of change, and that the *y*-intercept is zero when the function represents a proportional relationship.
			8.2.2.3	Identify how coefficient changes in the equation $f(x) = mx + b$ affect the graphs of linear functions. Know how to use graphing technology to examine these effects.
			8.2.2.4	Represent arithmetic sequences using equations, tables, graphs and verbal descriptions, and use them to solve problems. *For example*: If a girl starts with \$100 in savings and adds \$10 at the end of each month, she will have $100 + 10x$ dollars after x months.
			8.2.2.5	Represent geometric sequences using equations, tables, graphs and verbal descriptions, and use them to solve problems. *For example*: If a girl invests \$100 at 10% annual interest, she will have $100(1.1^x)$ dollars after x years.
8	Algebra	Generate equivalent numerical and algebraic expressions and use algebraic properties to evaluate expressions.	8.2.3.1	Evaluate algebraic expressions, including expressions containing radicals and absolute values, at specified values of their variables. *For example*: Evaluate $\pi r^2 h$ when $r = 3$ and $h = 0.5$, and then use an approximation of π, to obtain an approximate answer.
			8.2.3.2	Justify steps in generating equivalent expressions by identifying the properties used, including the properties of algebra. Properties include the associative, commutative and distributive laws, and the order of operations, including grouping symbols.
		Represent real-world and mathematical situations using equations and inequalities involving linear expressions. Solve equations and inequalities symbolically and graphically. Interpret solutions	8.2.4.1	Use linear equations to represent situations involving a constant rate of change, including proportional and non-proportional relationships. *For example*: For a cylinder with fixed radius of length 5, the surface area $A = 2\pi(5)h + 2\pi(5)^2 = 10\pi h + 50\pi$, is a linear function of the height h, but it is not proportional to the height.
			8.2.4.2	Solve multi-step equations in one variable. Solve for one variable in a multi-variable equation in terms of the other variables. Justify the steps by identifying the properties of equalities used. *For example*: The equation $10x + 17 = 3x$ can be changed to $7x + 17 = 0$, and then to $7x = -17$ by adding/subtracting the same quantities to both sides. These changes do not change the solution of the equation. *Another example*: Express the radius of a circle in terms of its circumference.

	Strand	Standard	No.	Benchmark
		in the original context.	8.2.4.3	Express linear equations in slope-intercept, point-slope and standard forms, and convert between these forms. Given sufficient information, find an equation of a line. *For example*: Determine an equation of the line through the points (-1,6) and (2/3, -3/4).
			8.2.4.4	Use linear inequalities to represent relationships in various contexts. *For example*: A gas station charges $0.10 less per gallon of gasoline if a customer also gets a car wash. Without the car wash, gas costs $2.79 per gallon. The car wash is $8.95. What are the possible amounts (in gallons) of gasoline that you can buy if you also get a car wash and can spend at most $35?
			8.2.4.5	Solve linear inequalities using properties of inequalities. Graph the solutions on a number line. *For example*: The inequality $-3x < 6$ is equivalent to $x > -2$, which can be represented on the number line by shading in the interval to the right of -2.
8	Algebra	Represent real-world and mathematical situations using equations and inequalities involving linear expressions. Solve equations and inequalities symbolically and graphically. Interpret solutions in the original context.	8.2.4.6	Represent relationships in various contexts with equations and inequalities involving the absolute value of a linear expression. Solve such equations and inequalities and graph the solutions on a number line. *For example*: A cylindrical machine part is manufactured with a radius of 2.1 cm, with a tolerance of 1/100 cm. The radius r satisfies the inequality $\|r - 2.1\| \le .01$.
			8.2.4.7	Represent relationships in various contexts using systems of linear equations. Solve systems of linear equations in two variables symbolically, graphically and numerically. *For example*: Marty's cell phone company charges $15 per month plus $0.04 per minute for each call. Jeannine's company charges $0.25 per minute. Use a system of equations to determine the advantages of each plan based on the number of minutes used.
			8.2.4.8	Understand that a system of linear equations may have no solution, one solution, or an infinite number of solutions. Relate the number of solutions to pairs of lines that are intersecting, parallel or identical. Check whether a pair of numbers satisfies a system of two linear equations in two unknowns by substituting the numbers into both equations.
			8.2.4.9	Use the relationship between square roots and squares of a number to solve problems. *For example*: If $\pi x^2 = 5$, then $\|x\| = \sqrt{\frac{5}{\pi}}$, or equivalently, $x = \sqrt{\frac{5}{\pi}}$ or $x = -\sqrt{\frac{5}{\pi}}$. If x is understood as the radius of a circle in this example, then the negative solution should be discarded and $x = \sqrt{\frac{5}{\pi}}$.
	Geometry & Measurement	Solve problems involving right triangles using the Pythagorean Theorem and its converse.	8.3.1.1	Use the Pythagorean Theorem to solve problems involving right triangles. *For example*: Determine the perimeter of a right triangle, given the lengths of two of its sides. *Another example*: Show that a triangle with side lengths 4, 5 and 6 is not a right triangle.
			8.3.1.2	Determine the distance between two points on a horizontal or vertical line in a coordinate system. Use the Pythagorean Theorem to find the distance between any two points in a coordinate system.

	Strand	Standard	No.	Benchmark
			8.3.1.3	Informally justify the Pythagorean Theorem by using measurements, diagrams and computer software.
		Solve problems involving parallel and perpendicular lines on a coordinate system.	8.3.2.1	Understand and apply the relationships between the slopes of parallel lines and between the slopes of perpendicular lines. Dynamic graphing software may be used to examine the relationships between lines and their equations.
8	Geometry & Measurement	Solve problems involving parallel and perpendicular lines on a coordinate system.	8.3.2.2	Analyze polygons on a coordinate system by determining the slopes of their sides. *For example*: Given the coordinates of four points, determine whether the corresponding quadrilateral is a parallelogram.
			8.3.2.3	Given a line on a coordinate system and the coordinates of a point not on the line, find lines through that point that are parallel and perpendicular to the given line, symbolically and graphically.
	Data Analysis & Probability	Interpret data using scatterplots and approximate lines of best fit. Use lines of best fit to draw conclusions about data.	8.4.1.1	Collect, display and interpret data using scatterplots. Use the shape of the scatterplot to informally estimate a line of best fit and determine an equation for the line. Use appropriate titles, labels and units. Know how to use graphing technology to display scatterplots and corresponding lines of best fit.
			8.4.1.2	Use a line of best fit to make statements about approximate rate of change and to make predictions about values not in the original data set. *For example*: Given a scatterplot relating student heights to shoe sizes, predict the shoe size of a 5'4" student, even if the data does not contain information for a student of that height.
			8.4.1.3	Assess the reasonableness of predictions using scatterplots by interpreting them in the original context. *For example*: A set of data may show that the number of women in the U.S. Senate is growing at a certain rate each election cycle. Is it reasonable to use this trend to predict the year in which the Senate will eventually include 1000 female Senators?
9, 10, 11	Algebra	Understand the concept of function, and identify important features of functions and other relations using symbolic and graphical methods.	9.2.1.1	Understand the definition of a function. Use functional notation and evaluate a function at a given point in its domain. *For example*: If $f(x)=\frac{1}{x^2-3}$, find $f(-4)$.
			9.2.1.2	Distinguish between functions and other relations defined symbolically, graphically or in tabular form.
			9.2.1.3	Find the domain of a function defined symbolically, graphically or in a real-world context. *For example*: The formula $f(x) = \pi x^2$ can represent a function whose domain is all real numbers, but in the context of the area of a circle, the domain would be restricted to positive x.

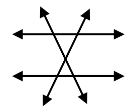
	Strand	Standard	No.	Benchmark
			9.2.1.4	Obtain information and draw conclusions from graphs of functions and other relations. *For example*: If a graph shows the relationship between the elapsed flight time of a golf ball at a given moment and its height at that same moment, identify the time interval during which the ball is at least 100 feet above the ground.
9, 10, 11	Algebra	Understand the concept of function, and identify important features of functions and other relations using symbolic and graphical methods where appropriate.	9.2.1.5	Identify the vertex, line of symmetry and intercepts of the parabola corresponding to a quadratic function, using symbolic and graphical methods, when the function is expressed in the form $f(x) = ax^2 + bx + c$, in the form $f(x) = a(x - h)^2 + k$, or in factored form.
			9.2.1.6	Identify intercepts, zeros, maxima, minima and intervals of increase and decrease from the graph of a function.
			9.2.1.7	Understand the concept of an asymptote and identify asymptotes for exponential functions and reciprocals of linear functions, using symbolic and graphical methods.
			9.2.1.8	Make qualitative statements about the rate of change of a function, based on its graph or table of values. *For example*: The function $f(x) = 3^x$ increases for all x, but it increases faster when $x > 2$ than it does when $x < 2$.
			9.2.1.9	Determine how translations affect the symbolic and graphical forms of a function. Know how to use graphing technology to examine translations. *For example*: Determine how the graph of $f(x) = \|x - h\| + k$ changes as h and k change.
		Recognize linear, quadratic, exponential and other common functions in real-world and mathematical situations; represent these functions with tables, verbal descriptions, symbols and graphs; solve	9.2.2.1	Represent and solve problems in various contexts using linear and quadratic functions. *For example*: Write a function that represents the area of a rectangular garden that can be surrounded with 32 feet of fencing, and use the function to determine the possible dimensions of such a garden if the area must be at least 50 square feet.
			9.2.2.2	Represent and solve problems in various contexts using exponential functions, such as investment growth, depreciation and population growth.
			9.2.2.3	Sketch graphs of linear, quadratic and exponential functions, and translate between graphs, tables and symbolic representations. Know how to use graphing technology to graph these functions.

	Strand	Standard	No.	Benchmark
		problems involving these functions, and explain results in the original context.	9.2.2.4	Express the terms in a geometric sequence recursively and by giving an explicit (closed form) formula, and express the partial sums of a geometric series recursively. *For example*: A closed form formula for the terms t_n in the geometric sequence 3, 6, 12, 24, ... is $t_n = 3(2)^{n-1}$, where $n = 1, 2, 3, \ldots$, and this sequence can be expressed recursively by writing $t_1 = 3$ and $t_n = 2t_{n-1}$, for $n \geq 2$. *Another example*: the partial sums s_n of the series 3 + 6 + 12 + 24 + ... can be expressed recursively by writing $s_1 = 3$ and $s_n = 3 + 2s_{n-1}$, for $n \geq 2$.
9, 10, 11	Algebra	Recognize linear, quadratic, exponential and other common functions in real-world and mathematical situations; represent these functions with tables, verbal descriptions, symbols and graphs; solve problems involving these functions, and explain results in the original context.	9.2.2.5	Recognize and solve problems that can be modeled using finite geometric sequences and series, such as home mortgage and other compound interest examples. Know how to use spreadsheets and calculators to explore geometric sequences and series in various contexts.
			9.2.2.6	Sketch the graphs of common non-linear functions such as $f(x)=\sqrt{x}$, $f(x)=\|x\|$, $f(x)=\frac{1}{x}$, $f(x) = x^3$, and translations of these functions, such as $f(x)=\sqrt{x-2}+4$. Know how to use graphing technology to graph these functions.
		Generate equivalent algebraic expressions involving polynomials and radicals; use algebraic properties to evaluate expressions.	9.2.3.1	Evaluate polynomial and rational expressions and expressions containing radicals and absolute values at specified points in their domains.
			9.2.3.2	Add, subtract and multiply polynomials; divide a polynomial by a polynomial of equal or lower degree.
			9.2.3.3	Factor common monomial factors from polynomials, factor quadratic polynomials, and factor the difference of two squares. *For example*: $9x^6 - x^4 = (3x^3 - x^2)(3x^3 + x^2)$.
			9.2.3.4	Add, subtract, multiply, divide and simplify algebraic fractions. *For example*: $\frac{1}{1-x}+\frac{x}{1+x}$ is equivalent to $\frac{1+2x-x^2}{1-x^2}$.

	Strand	Standard	No.	Benchmark
			9.2.3.5	Check whether a given complex number is a solution of a quadratic equation by substituting it for the variable and evaluating the expression, using arithmetic with complex numbers. *For example*: The complex number $\frac{1+i}{2}$ is a solution of $2x^2 - 2x + 1 = 0$, since $2\left(\frac{1+i}{2}\right)^2 - 2\left(\frac{1+i}{2}\right) + 1 = i - (1+i) + 1 = 0$.
9, 10, 11	Algebra	Generate equivalent algebraic expressions involving polynomials and radicals; use algebraic properties to evaluate expressions.	9.2.3.6	Apply the properties of positive and negative rational exponents to generate equivalent algebraic expressions, including those involving n^{th} roots. *For example*: $\sqrt{2} \times \sqrt{7} = 2^{\frac{1}{2}} \times 7^{\frac{1}{2}} = 14^{\frac{1}{2}} = \sqrt{14}$. Rules for computing directly with radicals may also be used: $\sqrt{2} \times \sqrt{x} = \sqrt{2x}$.
			9.2.3.7	Justify steps in generating equivalent expressions by identifying the properties used. Use substitution to check the equality of expressions for some particular values of the variables; recognize that checking with substitution does not guarantee equality of expressions for all values of the variables.
		Represent real-world and mathematical situations using equations and inequalities involving linear, quadratic, exponential, and nth root functions. Solve equations and inequalities symbolically and graphically. Interpret solutions in the original context.	9.2.4.1	Represent relationships in various contexts using quadratic equations and inequalities. Solve quadratic equations and inequalities by appropriate methods including factoring, completing the square, graphing and the quadratic formula. Find non-real complex roots when they exist. Recognize that a particular solution may not be applicable in the original context. Know how to use calculators, graphing utilities or other technology to solve quadratic equations and inequalities. *For example*: A diver jumps from a 20 meter platform with an upward velocity of 3 meters per second. In finding the time at which the diver hits the surface of the water, the resulting quadratic equation has a positive and a negative solution. The negative solution should be discarded because of the context.
			9.2.4.2	Represent relationships in various contexts using equations involving exponential functions; solve these equations graphically or numerically. Know how to use calculators, graphing utilities or other technology to solve these equations.
			9.2.4.3	Recognize that to solve certain equations, number systems need to be extended from whole numbers to integers, from integers to rational numbers, from rational numbers to real numbers, and from real numbers to complex numbers. In particular, non-real complex numbers are needed to solve some quadratic equations with real coefficients.
			9.2.4.4	Represent relationships in various contexts using systems of linear inequalities; solve them graphically. Indicate which parts of the boundary are included in and excluded from the solution set using solid and dotted lines.
			9.2.4.5	Solve linear programming problems in two variables using graphical methods.

	Strand	Standard	No.	Benchmark
9, 10, 11	Algebra	Represent real-world and mathematical situations using equations and inequalities involving linear, quadratic, exponential and nth root functions. Solve equations and inequalities symbolically and graphically. Interpret solutions in the original context.	9.2.4.6	Represent relationships in various contexts using absolute value inequalities in two variables; solve them graphically. *For example*: If a pipe is to be cut to a length of 5 meters accurate to within a tenth of its diameter, the relationship between the length x of the pipe and its diameter y satisfies the inequality $\|x - 5\| \le 0.1y$.
			9.2.4.7	Solve equations that contain radical expressions. Recognize that extraneous solutions may arise when using symbolic methods. *For example*: The equation $\sqrt{x-9} = 9\sqrt{x}$ may be solved by squaring both sides to obtain $x - 9 = 81x$, which has the solution $x = -\frac{9}{80}$. However, this is not a solution of the original equation, so it is an extraneous solution that should be discarded. The original equation has no solution in this case. *Another example*: Solve $\sqrt[3]{-x+1} = -5$.
			9.2.4.8	Assess the reasonableness of a solution in its given context and compare the solution to appropriate graphical or numerical estimates; interpret a solution in the original context.
	Geometry & Measurement	Calculate measurements of plane and solid geometric figures; know that physical measurements depend on the choice of a unit and that they are approximations.	9.3.1.1	Determine the surface area and volume of pyramids, cones and spheres. Use measuring devices or formulas as appropriate. *For example*: Measure the height and radius of a cone and then use a formula to find its volume.
			9.3.1.2	Compose and decompose two- and three-dimensional figures; use decomposition to determine the perimeter, area, surface area and volume of various figures. *For example*: Find the volume of a regular hexagonal prism by decomposing it into six equal triangular prisms.
			9.3.1.3	Understand that quantities associated with physical measurements must be assigned units; apply such units correctly in expressions, equations and problem solutions that involve measurements; and convert between measurement systems. *For example*: 60 miles/hour = 60 miles/hour × 5280 feet/mile × 1 hour/3600 seconds = 88 feet/second.
			9.3.1.4	Understand and apply the fact that the effect of a scale factor k on length, area and volume is to multiply each by k, k^2 and k^3, respectively.

	Strand	Standard	No.	Benchmark
9, 10, 11	Geometry & Measurement	Calculate measurements of plane and solid geometric figures; know that physical measurements depend on the choice of a unit and that they are approximations.	9.3.1.5	Make reasonable estimates and judgments about the accuracy of values resulting from calculations involving measurements. *For example*: Suppose the sides of a rectangle are measured to the nearest tenth of a centimeter at 2.6 cm and 9.8 cm. Because of measurement errors, the width could be as small as 2.55 cm or as large as 2.65 cm, with similar errors for the height. These errors affect calculations. For instance, the actual area of the rectangle could be smaller than 25 cm^2 or larger than 26 cm^2, even though 2.6 × 9.8 = 25.48.
		Construct logical arguments, based on axioms, definitions and theorems, to prove theorems and other results in geometry.	9.3.2.1	Understand the roles of axioms, definitions, undefined terms and theorems in logical arguments.
			9.3.2.2	Accurately interpret and use words and phrases in geometric proofs such as "if…then," "if and only if," "all," and "not." Recognize the logical relationships between an "if…then" statement and its inverse, converse and contrapositive. *For example*: The statement "If you don't do your homework, you can't go to the dance" is not logically equivalent to its inverse "If you do your homework, you can go to the dance."
			9.3.2.3	Assess the validity of a logical argument and give counterexamples to disprove a statement.
			9.3.2.4	Construct logical arguments and write proofs of theorems and other results in geometry, including proofs by contradiction. Express proofs in a form that clearly justifies the reasoning, such as two-column proofs, paragraph proofs, flow charts or illustrations. *For example*: Prove that the sum of the interior angles of a pentagon is 540˚ using the fact that the sum of the interior angles of a triangle is 180˚.
			9.3.2.5	Use technology tools to examine theorems, test conjectures, perform constructions and develop mathematical reasoning skills in multi-step problems. The tools may include compass and straight edge, dynamic geometry software, design software or Internet applets.
		Know and apply properties of geometric figures to solve real-world and mathematical problems and to logically justify results in geometry.	9.3.3.1	Know and apply properties of parallel and perpendicular lines, including properties of angles formed by a transversal, to solve problems and logically justify results. *For example*: Prove that the perpendicular bisector of a line segment is the set of all points equidistant from the two endpoints, and use this fact to solve problems and justify other results.

	Strand	Standard	No.	Benchmark
9, 10, 11	Geometry & Measurement	Know and apply properties of geometric figures to solve real-world and mathematical problems and to logically justify results in geometry.	9.3.3.2	Know and apply properties of angles, including corresponding, exterior, interior, vertical, complementary and supplementary angles, to solve problems and logically justify results. *For example*: Prove that two triangles formed by a pair of intersecting lines and a pair of parallel lines (an "X" trapped between two parallel lines) are similar. 
			9.3.3.3	Know and apply properties of equilateral, isosceles and scalene triangles to solve problems and logically justify results. *For example*: Use the triangle inequality to prove that the perimeter of a quadrilateral is larger than the sum of the lengths of its diagonals.
			9.3.3.4	Apply the Pythagorean Theorem and its converse to solve problems and logically justify results. *For example*: When building a wooden frame that is supposed to have a square corner, ensure that the corner is square by measuring lengths near the corner and applying the Pythagorean Theorem.
			9.3.3.5	Know and apply properties of right triangles, including properties of 45-45-90 and 30-60-90 triangles, to solve problems and logically justify results. *For example*: Use 30-60-90 triangles to analyze geometric figures involving equilateral triangles and hexagons. *Another example*: Determine exact values of the trigonometric ratios in these special triangles using relationships among the side lengths.
			9.3.3.6	Know and apply properties of congruent and similar figures to solve problems and logically justify results. *For example*: Analyze lengths and areas in a figure formed by drawing a line segment from one side of a triangle to a second side, parallel to the third side. *Another example*: Determine the height of a pine tree by comparing the length of its shadow to the length of the shadow of a person of known height. *Another example*: When attempting to build two identical 4-sided frames, a person measured the lengths of corresponding sides and found that they matched. Can the person conclude that the shapes of the frames are congruent?
			9.3.3.7	Use properties of polygons—including quadrilaterals and regular polygons—to define them, classify them, solve problems and logically justify results. *For example*: Recognize that a rectangle is a special case of a trapezoid. *Another example*: Give a concise and clear definition of a kite.
9, 10, 11	Geometry & Measurement	Know and apply properties of geometric figures to solve real-world and mathematical problems and to logically justify results in geometry.	9.3.3.8	Know and apply properties of a circle to solve problems and logically justify results. *For example*: Show that opposite angles of a quadrilateral inscribed in a circle are supplementary.

	Strand	Standard	No.	Benchmark
		Solve real-world and mathematical geometric problems using algebraic methods.	9.3.4.1	Understand how the properties of similar right triangles allow the trigonometric ratios to be defined, and determine the sine, cosine and tangent of an acute angle in a right triangle.
			9.3.4.2	Apply the trigonometric ratios sine, cosine and tangent to solve problems, such as determining lengths and areas in right triangles and in figures that can be decomposed into right triangles. Know how to use calculators, tables or other technology to evaluate trigonometric ratios. *For example*: Find the area of a triangle, given the measure of one of its acute angles and the lengths of the two sides that form that angle.
			9.3.4.3	Use calculators, tables or other technologies in connection with the trigonometric ratios to find angle measures in right triangles in various contexts.
			9.3.4.4	Use coordinate geometry to represent and analyze line segments and polygons, including determining lengths, midpoints and slopes of line segments.
			9.3.4.5	Know the equation for the graph of a circle with radius r and center (h,k), $(x-h)^2 + (y-k)^2 = r^2$, and justify this equation using the Pythagorean Theorem and properties of translations.
			9.3.4.6	Use numeric, graphic and symbolic representations of transformations in two dimensions, such as reflections, translations, scale changes and rotations about the origin by multiples of 90˚, to solve problems involving figures on a coordinate grid. *For example*: If the point (3,-2) is rotated 90˚ counterclockwise about the origin, it becomes the point (2,3).
			9.3.4.7	Use algebra to solve geometric problems unrelated to coordinate geometry, such as solving for an unknown length in a figure involving similar triangles, or using the Pythagorean Theorem to obtain a quadratic equation for a length in a geometric figure.
9, 10, 11	Data Analysis & Probability	Display and analyze data; use various measures associated with data to draw conclusions, identify trends and describe relationships.	9.4.1.1	Describe a data set using data displays, such as box-and-whisker plots; describe and compare data sets using summary statistics, including measures of center, location and spread. Measures of center and location include mean, median, quartile and percentile. Measures of spread include standard deviation, range and inter-quartile range. Know how to use calculators, spreadsheets or other technology to display data and calculate summary statistics.
			9.4.1.2	Analyze the effects on summary statistics of changes in data sets. *For example*: Understand how inserting or deleting a data point may affect the mean and standard deviation. *Another example*: Understand how the median and interquartile range are affected when the entire data set is transformed by adding a constant to each data value or multiplying each data value by a constant.
			9.4.1.3	Use scatterplots to analyze patterns and describe relationships between two variables. Using technology, determine regression lines (line of best fit) and correlation coefficients; use regression lines to make predictions and correlation coefficients to assess the reliability of those predictions.

	Strand	Standard	No.	Benchmark
			9.4.1.4	Use the mean and standard deviation of a data set to fit it to a normal distribution (bell-shaped curve) and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets and tables to estimate areas under the normal curve. *For example*: After performing several measurements of some attribute of an irregular physical object, it is appropriate to fit the data to a normal distribution and draw conclusions about measurement error. *Another example*: When data involving two very different populations is combined, the resulting histogram may show two distinct peaks, and fitting the data to a normal distribution is not appropriate.
		Explain the uses of data and statistical thinking to draw inferences, make predictions and justify conclusions.	9.4.2.1	Evaluate reports based on data published in the media by identifying the source of the data, the design of the study, and the way the data are analyzed and displayed. Show how graphs and data can be distorted to support different points of view. Know how to use spreadsheet tables and graphs or graphing technology to recognize and analyze distortions in data displays. *For example*: Shifting data on the vertical axis can make relative changes appear deceptively large.
			9.4.2.2	Identify and explain misleading uses of data; recognize when arguments based on data confuse correlation and causation.
			9.4.2.3	Explain the impact of sampling methods, bias and the phrasing of questions asked during data collection.
9, 10, 11	Data Analysis & Probability	Calculate probabilities and apply probability concepts to solve real-world and mathematical problems.	9.4.3.1	Select and apply counting procedures, such as the multiplication and addition principles and tree diagrams, to determine the size of a sample space (the number of possible outcomes) and to calculate probabilities. *For example*: If one girl and one boy are picked at random from a class with 20 girls and 15 boys, there are $20 \times 15 = 300$ different possibilities, so the probability that a particular girl is chosen together with a particular boy is $\frac{1}{300}$.
			9.4.3.2	Calculate experimental probabilities by performing simulations or experiments involving a probability model and using relative frequencies of outcomes.
			9.4.3.3	Understand that the Law of Large Numbers expresses a relationship between the probabilities in a probability model and the experimental probabilities found by performing simulations or experiments involving the model.
			9.4.3.4	Use random numbers generated by a calculator or a spreadsheet, or taken from a table, to perform probability simulations and to introduce fairness into decision making. *For example*: If a group of students needs to fairly select one of its members to lead a discussion, they can use a random number to determine the selection.
			9.4.3.5	Apply probability concepts such as intersections, unions and complements of events, and conditional probability and independence, to calculate probabilities and solve problems. *For example*: The probability of tossing at least one head when flipping a fair coin three times can be calculated by looking at the complement of this event (flipping three tails in a row).

	Strand	Standard	No.	Benchmark
			9.4.3.6	Describe the concepts of intersections, unions and complements using Venn diagrams. Understand the relationships between these concepts and the words AND, OR, NOT, as used in computerized searches and spreadsheets.
			9.4.3.7	Understand and use simple probability formulas involving intersections, unions and complements of events. *For example*: If the probability of an event is p, then the probability of the complement of an event is $1 - p$; the probability of the intersection of two independent events is the product of their probabilities. *Another example*: The probability of the union of two events equals the sum of the probabilities of the two individual events minus the probability of the intersection of the events.
			9.4.3.8	Apply probability concepts to real-world situations to make informed decisions. *For example*: Explain why a hockey coach might decide near the end of the game to pull the goalie to add another forward position player if the team is behind. *Another example*: Consider the role that probabilities play in health care decisions, such as deciding between having eye surgery and wearing glasses.
9, 10, 11	Data Analysis & Probability	Calculate probabilities and apply probability concepts to solve real-world and mathematical problems.	9.4.3.9	Use the relationship between conditional probabilities and relative frequencies in contingency tables. *For example*: A table that displays percentages relating gender (male or female) and handedness (right-handed or left-handed) can be used to determine the conditional probability of being left-handed, given that the gender is male.